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DEFICIENCY IN INFINITE-DIMENSIONAL MANIFOLDS
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1. Introduction. In [17] it was shown that if E is a metric topological vector space (MTVS) which is homeomorphic (\cong) to its own countable infinite product E^ω , and M is any E-manifold (i.e. M is a paracompact manifold modeled on E), then $M \cong M \times E$. Accordingly we define a subset K of M to have E-deficiency (or to be E-deficient) provided that K is closed and there exists a homeomorphism $h: M \rightarrow M \times E$ such that $h(K) \subset M \times \{0\}$.

Such sets have proved to be important to the point-set topology of infinite dimensional manifolds because of results of the following two types.

(1) Negligibility theorem. If ℓ^2 is separable infinite-dimensional Hilbert space, M is an ℓ^2 -manifold, and $K \subset M$ is a countable union of ℓ^2 -deficient sets, then it was shown in [1] that $M \cong M \setminus K$. More general results have been established in [8].

(2) Homeomorphism extension theorems. If M is as in (1), K_1 and K_2 are ℓ^2 -deficient sets in M , and $h: K_1 \rightarrow K_2$ is a homeomorphism which is homotopic to id_{K_1} (the identity on K_1), then h can be extended to a manifold homeomorphism [6]. For K_1 and K_2 additionally assumed to be ANR's, a similar result has been established for more general linear spaces than ℓ^2 [11].

In applications it is not easy to recognize that some sets have E-deficiency, thus it becomes desirable to have a coordinate-free topological characterization of E-deficiency in E-manifolds M . Such a characterization was obtained in [3] for $M = \ell^2$ and in [7] it was generalized to M any ℓ^2 -manifold. It states (using a notion introduced by Anderson in [3]) that $K \subset M$ (where M is an ℓ^2 -manifold) is ℓ^2 -deficient iff K has Property Z (or is a Z-set), where a set F in a space X has Property Z iff F is closed and for each non-null, homotopically trivial open set U in X , $U \setminus F$ is non-null and homotopically trivial. Among other things this enables us to recognize collared, closed sub-manifolds of M (i.e. boundaries of M) as being ℓ^2 -deficient and any

closed subset of M which is a countable union of ℓ^2 -deficient sets is itself ℓ^2 -deficient.

The main result of this paper is the following, which generalizes this characterization of E -deficiency to Fréchet manifolds.

Theorem 1. Let $E \cong E^\omega$ be a Fréchet space, M be an E -manifold, and let $K \subset M$. Then K has E -deficiency iff K has Property Z .

We remark that there are no known examples of infinite-dimensional Fréchet spaces E which do not satisfy the condition $E \cong E^\omega$.

Concerning techniques it should be remarked that the proof of the corresponding result for $M = \ell^2$ [3] used the topology of the Hilbert cube I^∞ and the fact that ℓ^2 can be compactified by I^∞ (since ℓ^2 is homeomorphic to the countable product of lines [4]). The proof we give for Theorem 1 also uses the fact that ℓ^2 can be compactified by I^∞ .

Using Theorem 1 above and Theorem 1 of [8] we easily obtain the following result.

Corollary. Let M be as in Theorem 1 and let $K \subset M$ be a countable union of Z -sets. Then K is strongly negligible in M , i.e. there exists a homeomorphism $h: M \rightarrow M \setminus K$ which may be chosen arbitrarily close to id_M .

(The notion of "arbitrarily close" will be made precise in the next section). We remark that by using different techniques David W. Henderson has recently shown (unpublished) that single Z -sets are strongly negligible in E -manifolds, where $E \cong E^\omega$ is a locally convex (LC) MTVS.

In Theorem 2 we establish a homeomorphism extension theorem which generalizes the extension theorem of [6] (which was proved for ℓ^2 -manifolds). In Theorem 2' below we give a simplified version of Theorem 2. The more general statement appears in Section 5.

Theorem 2'. Let $E \cong E^\omega$ be a LCMTVS and let M be an E -manifold. If K_1 and K_2 are E -deficient subsets of M and $h: K_1 \rightarrow K_2$ is a homeomorphism which is homotopic to id_{K_1} , then h can be extended to a manifold homeomorphism.

2. Preliminaries. In this paper all spaces will be assumed to be metric and all homeomorphisms will be assumed to be onto.

Let X and Y be spaces and let \mathcal{U} be an open cover of Y . Then functions $f, g: X \rightarrow Y$ are said to be \mathcal{U} -close provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $f(x), g(x) \in U$. A function $F: X \times I \rightarrow Y$ (where $I = [0, 1]$) is said to be limited by \mathcal{U} provided that for each $x \in X$ there exists a $U \in \mathcal{U}$ such that $F(\{x\} \times I) \subset U$.

If X is a space and $F \subset X$ is closed, then by Lemma 3 of [5] there exists an open cover \mathcal{U} of $X \setminus F$ such that if $h: X \setminus F \rightarrow X \setminus F$ is any homeomorphism which is \mathcal{U} -close to $\text{id}_{X \setminus F}$, then h can be extended to a homeomorphism $\tilde{h}: X \rightarrow X$ which satisfies $\tilde{h}|_F = \text{id}_F$. Such a cover of $X \setminus F$ will be called normal (with respect to F).

Let X be a space and let $\{f_i\}_{i=1}^{\infty}$ be a collection of homeomorphisms of X onto itself. Then for each $x \in X$ we let $f(x) = \lim_{i \rightarrow \infty} f_i \circ \dots \circ f_1(x)$, if this limit exists. If $f(x)$ exists, for all $x \in X$, then we write $f = \text{L}\Pi_{i=1}^{\infty} f_i$, and call it the infinite left product of $\{f_i\}_{i=1}^{\infty}$. We now state a convergence criterion for infinite left products. This is essentially a reformulation of West's version [18] of Theorem 4.2 of [4].

Convergence Procedure. Let X be a (topologically) complete space and let \mathcal{U} be an open cover of X . Then to each homeomorphism $f: X \rightarrow X$ and each integer $i > 0$ we can assign an open cover $\mathcal{U}_i(f)$ of X such that if $\{f_i\}_{i=1}^{\infty}$ is any collection of homeomorphisms of X onto itself for which f_{i+1} is $\mathcal{U}_i(f_i \circ \dots \circ f_1)$ -close to id_X , for all $i > 0$, then $f = \text{L}\Pi_{i=1}^{\infty} f_i$ gives a homeomorphism of X onto itself which is \mathcal{U} -close to id_X .

There is one other notion of deficiency which will be useful in the sequel. Let X be a space which is homeomorphic to $X \times \ell^2$ and let $K \subset X$. Then K has ℓ^2 -deficiency provided that K is closed and there exists a homeomorphism $h: X \rightarrow X \times \ell^2$ which satisfies $h(K) \subset X \times \{0\}$.

We will represent the Hilbert cube I^{∞} as $\Pi_{i=1}^{\infty} I_i$, where each I_i is the closed interval $[-1, 1]$. The set $\Pi_{i=1}^{\infty} I_i^{\circ}$, where $I_i^{\circ} = (-1, 1)$, will be denoted by s . In [2] it is shown that $s \times I^{\infty} \cong s$ and we have already remarked that $s \cong \ell^2$.

3. The equivalence of E- and ℓ^2 -deficiency. The main result of this section is Theorem 3.1, where we show that in certain spaces E-deficiency and ℓ^2 -deficiency are equivalent concepts. A similar proposition was established in [9], where E was additionally assumed to be a Banach space. We remark that the proof we give of Theorem 3.1 below follows in broad outline the proof of the corresponding result of [9], with appropriate modifications being made to overcome the lack of a norm. We will first need a technical result concerning open cones. (By the open cone over a space X (denoted by $C(X)$) we mean the space $\{v\} \cup (X \times (0,1))$, which is topologized by choosing as a basis the usual topology on $X \times (0,1)$ together with all sets of the form $\{v\} \cup (X \times (0,t))$, for all $t \in (0,1)$. We call v the vertex of the cone). We omit the proof of the lemma, since it is similar to Theorem 5.3 of [17].

Lemma 3.1. Let $E \cong E^\omega$ be a MTVS. Then there exists a homeomorphism $h: E \times [0,1) \times E \rightarrow C(E) \times E$ which satisfies the following properties.

- (1) $h(E \times \{t\} \times E) = E \times \{t\} \times E$, for all $t \in (0,1)$,
- (2) $h(E \times \{0\} \times E) = \{v\} \times E$, where v is the vertex of $C(E)$.

Theorem 3.1. Let $E \cong E^\omega$ be a MTVS, M be an E-manifold, and let $K \subset M$. Then K has E-deficiency iff K has ℓ^2 -deficiency.

Proof. Assume K has E-deficiency and let $f: M \rightarrow M \times E \times E$ be a homeomorphism such that $f(K) \subset M \times E \times \{0\}$. By the Bartle-Graves-Michael Theorem [15] we have $E \cong (-1,1) \times G$, for some G . Thus

$$E \cong E^\omega \cong (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times (-1,1)^\omega \times G^\omega \cong (-1,1)^\omega \times E.$$

Since $(-1,1)^\omega \cong \ell^2$ we have $E \cong E \times \ell^2$. Let $g: E \rightarrow E \times \ell^2$ be a homeomorphism which satisfies $g(0) = (0,0)$ and let $\tilde{f}: M \rightarrow M \times E \times E \times \ell^2$ be defined by $\tilde{f}(x) = (id \times g) \circ f(x)$, where $id \times g: M \times E \times E \rightarrow M \times E \times E \times \ell^2$ is defined by $(id \times g)(x,y,z) = (x,y,g(z))$. Then \tilde{f} is a homeomorphism and $\tilde{f}(K) \subset M \times E \times E \times \{0\}$. This implies that K has ℓ^2 -deficiency.

On the other hand assume that K has ℓ^2 -deficiency. Thus we have a homeomorphism $f: M \rightarrow M \times \ell^2$ such that $f(K) \subset M \times \{0\}$. It is easy to modify f to get a homeomorphism $\tilde{f}: M \rightarrow M \times E \times [0,1) \times E$ which satisfies

$\tilde{f}(K)CM \times E \times \{0\} \times E$. (Use the fact that $\ell^2 \cong \ell^2 \times [0,1)$ [13]). Let $h: E \times [0,1) \times E \rightarrow C(E) \times E$ be the homeomorphism described in Lemma 3.1. Then $\text{id}_M \times h: M \times E \times [0,1) \times E \rightarrow M \times C(E) \times E$ is a homeomorphism and $(\text{id}_M \times h) \circ \tilde{f}(K)CM \times \{v\} \times E$, where v is the vertex of $C(E)$.

In the proof of Lemma 2 of [12] there is a proof that $E \times \ell^2 \cong C(E \times S_1)$, where $S_1 = \{x \in \ell^2 \mid \|x\| = 1\}$. Since $S_1 \cong \ell^2$ [13] we have $E \cong C(E)$. Thus we can modify $(\text{id}_M \times h) \circ \tilde{f}$ to get a homeomorphism $g: M \rightarrow M \times E$ which satisfies $g(K) \subset M \times \{0\}$. \square

4. Deforming a manifold to an ℓ^2 -deficient subset. The main result of this section is Theorem 4.2, which shows how to deform certain manifolds onto subsets which have ℓ^2 -deficiency. The following lemma is needed for its proof.

Lemma 4.1. Let X be a space and let K be a closed subset of $X \times I^\infty$ such that $K \subset X \times s$. Then there exists a homeomorphism $f: X \times I^\infty \rightarrow X \times I^\infty$ which satisfies $f(K) \subset X \times \prod_{i=1}^\infty [-\frac{1}{2}, \frac{1}{2}]$.

Proof. For each integer $i > 0$ and each $x \in X$ let

$$f_i(x) = \begin{cases} \text{glb } \{t_i \mid (x, t) \in K\}, & \text{if } K \cap (\{x\} \times I^\infty) \neq \emptyset \\ 1, & \text{if } K \cap (\{x\} \times I^\infty) = \emptyset \end{cases}$$

where we adopt the convention that if $t \in I^\infty$, then t_i is the i^{th} coordinate of t . It follows routinely that each $f_i: X \rightarrow (-1, 1]$ is lower semi-continuous. Thus by Dowker's theorem ([10], page 170) there is a continuous function $g_i: X \rightarrow (-1, 1)$ which satisfies $-1 < g_i(x) < f_i(x)$, for all $x \in X$. Similarly there is a continuous function $g_i^1: X \rightarrow (-1, 1)$ which satisfies $-1 < g_i(x) < g_i^1(x) < 1$ and $\tau_i \circ \pi_{I^\infty}(K \cap (\{x\} \times I^\infty)) \subset (g_i(x), g_i^1(x))$, for all $x \in X$, where $\tau_i: I^\infty \rightarrow I_i$ and $\pi_{I^\infty}: X \times I^\infty \rightarrow I^\infty$ are projections.

For each pair a, b , of real numbers satisfying $-1 < a < b < 1$, there exists a unique piecewise linear homeomorphism $h_{a,b}: [-1, 1] \rightarrow [-1, 1]$ which satisfies $h_{a,b}(a) = -\frac{1}{2}$, $h_{a,b}(b) = \frac{1}{2}$, and $h_{a,b}$ is linear on each of the intervals $[-1, a]$, $[a, b]$, and $[b, 1]$.

Then define $f: X \times I^\infty \rightarrow X \times I^\infty$ by $f(x, (t_i)) = (x, (h_{g_i(x), g_i^1(x)}(t_i)))$.

for all $(x, (t_i)) \in X \times I^\infty$. Clearly f fulfills our requirements. \square

Theorem 4.1. Let $E \cong E^\omega$ be a MTVS, M be an E -manifold, $K \subset M$ be ℓ^2 -deficient, and let \mathcal{U} be an open cover of M . Then there exists a homotopy $H: M \times I \rightarrow M$ such that $H_0 = \text{id}$, $H_t|_K = \text{id}$, for all $t \in I$, $H_t: M \rightarrow M$ is an embedding such that $H_t(M)$ is ℓ^2 -deficient, and H is limited by \mathcal{U} .

Proof. Since $I^\infty \times s \cong s$ we can use an argument like that used in Lemma 6 of [7] to prove that a closed set $F \subset M$ has ℓ^2 -deficiency iff there exists a homeomorphism of M onto $M \times I^\infty$ taking F into $M \times \{0\}$. Thus let $f: M \rightarrow M \times I^\infty$ be a homeomorphism such that $f(K) \subset M \times \{0\}$.

Using techniques like those used in Lemma 4.1 we can clearly construct a homotopy $G: M \times I^\infty \times I \rightarrow M \times I^\infty$ such that $G_0 = \text{id}$, $G_t: M \times I^\infty \rightarrow M \times I^\infty$ is a closed embedding such that $G_t(M \times I^\infty) \subset M \times s$, $G_t|_{M \times \{0\}} = \text{id}$, for all t , and G is limited by $f(\mathcal{U})$, the cover of $M \times I^\infty$ induced by f and \mathcal{U} . Then define $H: M \times I \rightarrow M$ by $H_t(x) = f^{-1} \circ G_t \circ f(x)$. All we have to do is show that $H_t(M)$ has ℓ^2 -deficiency.

Note that $f \circ H_t(M)$ is a closed subset of $M \times I^\infty$ which satisfies $f \circ H_t(M) \subset M \times s$. Using Lemma 4.1 there exists a homeomorphism $g: M \times I^\infty \rightarrow M \times I^\infty$ such that $g \circ f \circ H_t(M) \subset \prod_{i=1}^\infty [-\frac{1}{2}, \frac{1}{2}]$. From [2] it follows that there exists a homeomorphism $h: I^\infty \rightarrow I^\infty \times I^\infty$ such that $h(\prod_{i=1}^\infty [-\frac{1}{2}, \frac{1}{2}]) \subset I^\infty \times \{0\}$. Let $\tilde{h}: M \times I^\infty \rightarrow M \times I^\infty \times I^\infty$ be defined by $\tilde{h}(x, y) = (x, h(y))$. Then $\tilde{h} \circ g \circ f \circ H_t(M) \subset M \times I^\infty \times \{0\}$. By our comments above this proves that $H_t(M)$ has ℓ^2 -deficiency. \square

5. A proof of Theorem 2. The main result of this section is Theorem 2, where we generalize the homeomorphism extension theorem of [6]. The proof we give follows in broad outline the proof given in [6], but there are a few technicalities which have to be overcome in order to make the proof work. We will need the following mapping replacement theorem which resembles Theorem 3.1 of [6].

Lemma 5.1. Let $E \cong E^\omega$ be a LCMTVS, M be an E -manifold, X be a space which can be embedded as a closed subset of E , ACX be closed, and let $f: X \rightarrow M$ be a continuous function such that $f|_A$ is a homeomorphism of A onto an E -deficient subset of M . If \mathcal{U} is any open cover of M , then there exists an embedding $g: X \rightarrow M$ such that $g(X)$ is E -deficient, $g|_A = f|_A$, and g is \mathcal{U} -close to f .

Proof. Using Theorem 4.1 and the fact that E is an AR [16], a proof can be given which is similar to Theorem 3.1 of [6]. \square

We will also need the following generalization of Theorem 2 of [7].

Lemma 5.2. Let $E \cong E^\omega$ be a MTVS, M be a connected E -manifold, and let KCM be an E -deficient set. Then M can be embedded as an open subset of E so that K is taken onto an E -deficient (and therefore closed) subset of E .

Proof. The proof proceeds routinely as in Theorem 2 of [7] provided we note that (1) M can be embedded as an open subset of E , and (2) there exists a homotopy $H: E \times I \rightarrow E$ such that $H_0 = \text{id}_E$, H_t is a homeomorphism (onto), for $0 \leq t < 1$, and $H_1: E \rightarrow E \setminus \{0\}$ is a homeomorphism. The first assertion is just Theorem 4 of [12] and the second assertion follows since a corresponding property is true for ℓ^2 [4] and also since $E \cong E \times \ell^2$ (as was noted in Theorem 3.1). \square

Theorem 2. Let $E \cong E^\omega$ be a LCMTVS, M be an E -manifold, and let K_1, K_2 be E -deficient subsets of M for which there exists a homotopy $H: K_1 \times I \rightarrow M$ such that $H_0 = \text{id}_{K_1}$ and $H_1: K_1 \rightarrow K_2$ is a homeomorphism. If \mathcal{U} is an open cover of M such that H is limited by \mathcal{U} , then there exists an ambient

invertible isotopy $G : M \times I \rightarrow M$ which satisfies $G_0 = \text{id}_M$, $G_1|_{K_1} = H_1$, and G is limited by $\text{St}^3(\mathcal{U})$ (the 3rd star of the cover \mathcal{U}).

(An isotopy $G : X \times I \rightarrow X$ is said to be an ambient invertible isotopy provided that each level is an onto homeomorphism and $G^* : X \times I \rightarrow X$, defined by $G_t^*(x) = G_t^{-1}(x)$, is continuous).

Proof. First note that a homeomorphism extension theorem for E (without the limitation by covers) is easy to establish for E -deficient subsets of E . One merely uses the technique of Klee [14], as used in [2]. Thus in the case that $K_1 \cap K_2 = \emptyset$ we can use Lemma 5.1 and the techniques of [6] to obtain our desired ambient invertible isotopy.

On the other hand assume that $K_1 \cap K_2 \neq \emptyset$. It follows routinely that K_1 and K_2 are Z -sets, and therefore $K_1 \cup K_2$ is a Z -set. Using an unpublished result of David W. Henderson there exists, for each open cover \mathcal{U}' of M , a homotopy $F : M \times I \rightarrow M$ such that $F_0 = \text{id}_M$, $\text{Cl}(F_1(M)) \cap (K_1 \cup K_2) = \emptyset$ (where Cl denotes closure), and F is limited by \mathcal{U}' . Thus by Lemma 5.1 and an appropriate choice of \mathcal{U}' , there exists an embedding $F^* : K_2 \times I \rightarrow M$ such that $F_0^* = \text{id}_{K_2}$, $F^*(K_2 \times I)$ is an E -deficient set in M for which $F_1^*(K_2) \cap (K_1 \cup K_2) = \emptyset$, and F^* is limited by \mathcal{U} .

Using the above remarks there exists an ambient invertible isotopy $G^* : M \times I \rightarrow M$ such that $G_0^* = \text{id}_M$, $G_1^*|_{K_2} = F_1^*$, and G^* is limited by \mathcal{U} . Note that K_1 and $F_1^*(K_2)$ are disjoint E -deficient subsets of M and $F_1^* \circ H_1 : K_1 \rightarrow F_1^*(K_2)$ is a homeomorphism which is homotopic to id_{K_1} , with a homotopy that is limited by $\text{St}(\mathcal{U})$. We can once more use the above techniques to find an ambient invertible isotopy $H^* : M \times I \rightarrow M$ such that $H_0^* = \text{id}_M$, $H_1^*|_{K_1} = F_1^* \circ H_1$, and H^* is limited by $\text{St}^2(\mathcal{U})$. Then the obvious composition $G = (G^*)^{-1} \circ H^*$ fulfills our requirements. \square

6. Proof of Theorem 1. The step from E-deficiency to Property Z is straightfoward and resembles Theorem 9.1 of [3]. For the other implication let $K \subset M$ have Property Z and let $h : M \rightarrow M \times I^\infty$ be a homeomorphism. Using the representation for I^∞ and s given in Section 2 let $B(I^\infty) = I^\infty \setminus s$. It is shown in [3] that there is a homeomorphism of I^∞ onto itself which sends $B(I^\infty)$ into s . We can obviously write $B(I^\infty) = \bigcup_{n=1}^\infty C_n$, where each C_n is compact. Thus using the above comment and the techniques of Theorem 4.1 it follows that each $M \times C_n$ is E-deficient in $M \times I^\infty$.

We will describe a sequence $\{g_i\}_{i=1}^\infty$ of homeomorphisms of $M \times I^\infty$ onto itself whose left product $g = \prod_{i=1}^\infty g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself which satisfies $g \circ h(K) \subset M \times s$. Then we can apply the techniques of Theorem 4.1 to conclude that $g \circ h(K)$ is E-deficient.

Since $h(K)$ is a Z-set in $M \times I^\infty$ we can use the technique of the proof of Theorem 2 to get a homeomorphism $g_1 : M \times I^\infty \rightarrow M \times I^\infty$ such that $g_1 \circ h(K) \cap (M \times C_1) = \emptyset$. Now invoking the Convergence Procedure of Section 2 we need to produce a homeomorphism $g_2 : M \times I^\infty \rightarrow M \times I^\infty$ which is \mathcal{U} -close to $\text{id}_{M \times I^\infty}$, for any prechosen open cover \mathcal{U} of $M \times I^\infty$, and $g_2 \circ g_1 \circ h(K) \cap (M \times (C_1 \cup C_2)) = \emptyset$. Once more using the fact that $M \times (C_1 \cup C_2)$ is E-deficient and $g_1 \circ h(K)$ is a Z-set, we can use the techniques of Theorem 2 to obtain the desired g_2 .

Thus using an inductive procedure we can choose homeomorphisms $g_i : M \times I^\infty \rightarrow M \times I^\infty$ so that $g_i \circ \dots \circ g_1 \circ h(K) \cap (M \times (\bigcup_{n=1}^i C_n)) = \emptyset$ and $g = \prod_{i=1}^\infty g_i$ gives a homeomorphism of $M \times I^\infty$ onto itself. Since we are able to select each g_i arbitrarily close to $\text{id}_{M \times I^\infty}$ we can choose $\{g_i\}_{i=1}^\infty$ so that $g \circ h(K) \cap (M \times B(I^\infty)) = \emptyset$, thus $g \circ h(K) \subset M \times s$ and we are done. \square

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